Applications / Data Bulletin 175

GROUP DELAY EQUALIZATION IN COMMUNICATIONS SYSTEMS

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Group Delay Equalization in Communications Systems

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I. INTRODUCTION

The purpose of this Applications Bulletin is to introduce the reader to the general problem of group delay compensation of linear networks. The effects of group delay distortion on the time domain response of a linear network are discussed, as well as methods for producing sharp cutoff filters with linear phase characteristics. In addition, computer optimization and equalizer parameters are introduced with the viewpoint of intelligently specifying cost-effective equalizers and equalized filters.

It is not the intention of the authors to delve into the attributes or design of specific communications systems as they pertain to group delay distortion, but instead to present guidelines that permit system designers to specify filters and equalizers to the present state of the art.

II. GROUP DELAY AND ITS DEFINITION

The purpose of frequency dependent elements in any communications system is to provide signal-to-noise ratio enhancement. This is generally achieved by relating a band limited signal f(t) to its frequency spectrum $F(\omega)$, passing the frequencies contained in $F(\omega)$, and rejecting those frequencies not contained in $F(\omega)$. Naturally, it is hoped that f(t) can be recovered with as little distortion as possible. Historically, communications systems have been concerned with voice transmission; the final receiver being the human ear. The physiology of the human ear is such that the frequency content of the signal is far more important than the relative phase of those frequencies. Since the phase relationship was of minor importance to the "undistorted" transmission of voice, filters with sharp cutoff frequencies could be utilized thus limiting noise components without regard to the phase characteristics of the filter.

The advent and burgeoning of data communications has altered these basic precepts. We now find that the phase characteristics of filters is as important as its amplitude characteristics—sometimes more so.

Communication channels possess phase shift vs. frequency characteristics typified by Figure 1.

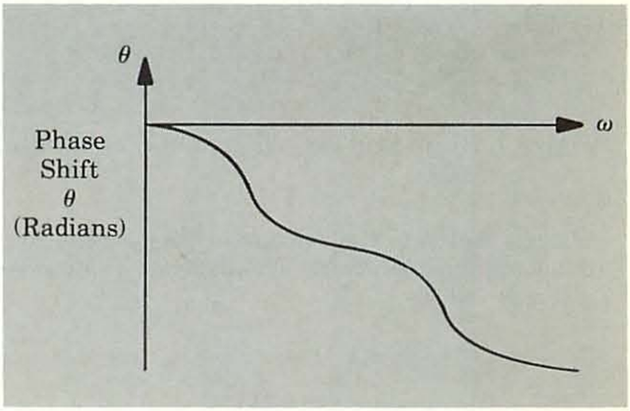


Figure 1. Typical Phase Shift of a Communications Channel

The slope of the phase characteristic is defined as the group (or envelope) delay, specifically:

$$\text{Group Delay} = \frac{-d\theta}{d\omega} = G_d \text{ the derivative of phase.}$$

Figure 2 shows the group delay corresponding to the phase characteristics shown in Figure 1.

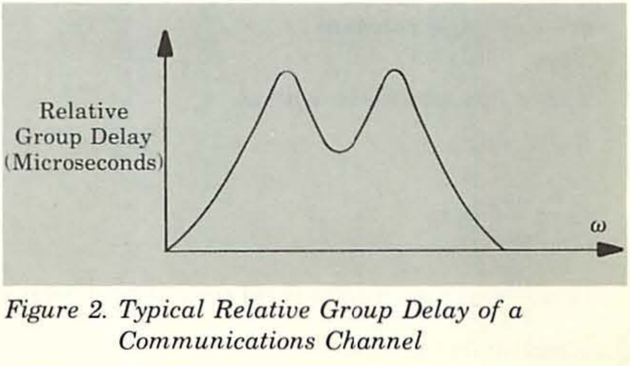


Figure 2. Typical Relative Group Delay of a Communications Channel

It is the *relative difference* of the group delay within the channel that we call group delay distortion. If the phase shift of a system were linear, the group delay would be constant and the group delay distortion would then be zero. (See Figure 3) This, of course, would be the group delay of an ideal system.

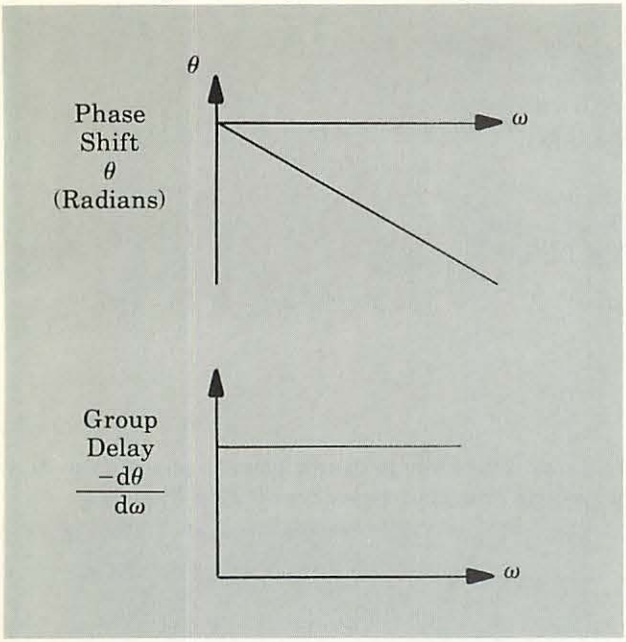


Figure 3. Phase Shift and Group Delay of an Ideal System

We can motivate the concept of group delay by the following considerations: The delay of a sine wave is given as follows:

if $f(t) = \cos \omega_0 t$

then $f(t)_{(\text{delayed})} = \cos \omega_0 (t - t_o)$

where t_o is the delay in time.

Thus, if we consider the problem of passing $F(\omega)$ through some transfer function $|A(\omega)|\epsilon^{j\theta(\omega)}$ with minimum distortion, it is apparent that $A(\omega)$ should be constant and that $\theta(\omega)$ should be such that the frequency components "line up" exactly as they did in $F(\omega)$. That is to say, if one frequency component experiences a delay of t_o through $A(\omega)$, then all of the frequency components of $F(\omega)$ must experience the same delay.

or $t_o = \text{constant}$

$\omega_o t_o = \theta_o$

and θ_o must be linear with ω_o

if $\omega_o t_o = \theta_o$

then $\dfrac{d\theta_o}{d\omega_o} = t_o$

thus $\dfrac{d\theta_o}{d\omega_o}$ is a good measure of constancy of delay over a band of frequencies.

This concept of a measure of constant delay over a band of frequencies is further motivated by the following considerations:

Assume an amplitude modulated sinewave of the form:

$f(t) = f_1(t) \cos \omega_o t$

where $f_1(t)$ has a Fourier Transform $F_1(\omega)$

$F_1(\omega) \rightleftarrows f_1(t)$

Also assume that $f_1(t)$ is band limited

$F_1(\omega) = 0 \qquad \text{for all } |\omega| > \Omega$

If for $(\omega_o - \Omega, \omega_o + \Omega)$; $|A(\omega)|$ is constant, and $\theta(\omega)$ is linear, then

$A(\omega) = A(\omega_o); \omega_o - \Omega < \omega < \omega_o + \Omega$

and

$\theta(\omega) = \theta(\omega_o) + \theta'(\omega_o)(\omega - \omega_o) = \omega_o t_{ph}(\omega_o) + (\omega - \omega_o) t_{gr}(\omega_o)$

where t_{ph} is phase delay and t_{gr} is group delay.

This network, therefore, acts as an ideal symmetric band pass filter and the output $g(t)$ is given by:

$g(t) = f_1(t - t_{gr}) \cos \omega_o (t - t_{ph})$

where we assume $A(\omega_o) = 1$

Note that $t_{gr}(\omega - \omega_o)$ = delay of envelope of $f_1(t)$ and t_{ph} = delay of carrier.

In a physical system $A(\omega)$ and $\theta(\omega)$ are not constant and may not be symmetric. If this variation is small, then t_{gr} gives the displacement of the center of gravity of the envelope and can be referred to as Group Delay. Note that we require a relatively narrow band system. $t_{gr}(\omega)$ gives a measure of the deviation from linear phase—a specification we can label "variation of group delay". (See Figure 4)

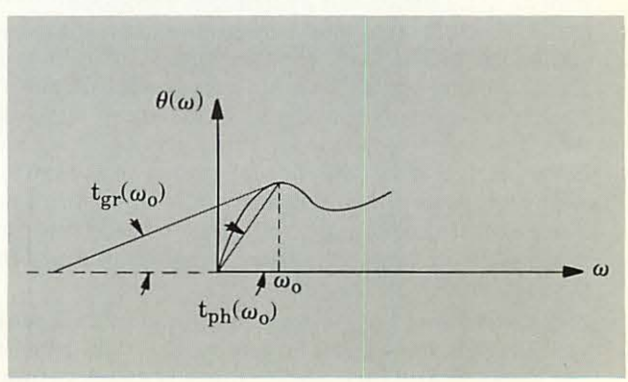


Figure 4. Variation of Group Delay

Although strictly speaking group delay is not an attribute of low pass filters, it is common practice to specify variation of group delay for low pass filters as a measure of phase linearity. This practice has arisen for good reason.

Firstly, in communications systems, it is common to mix baseband signals up or down with respect to an IF frequency. Within the context of the IF frequency (usually relatively narrow band), group delay is well defined and the group delay of the low pass filter adds to group delay of the IF filter, as we will subsequently show.

Secondly, it is generally easier and preferable to equalize IF networks at baseband frequencies.

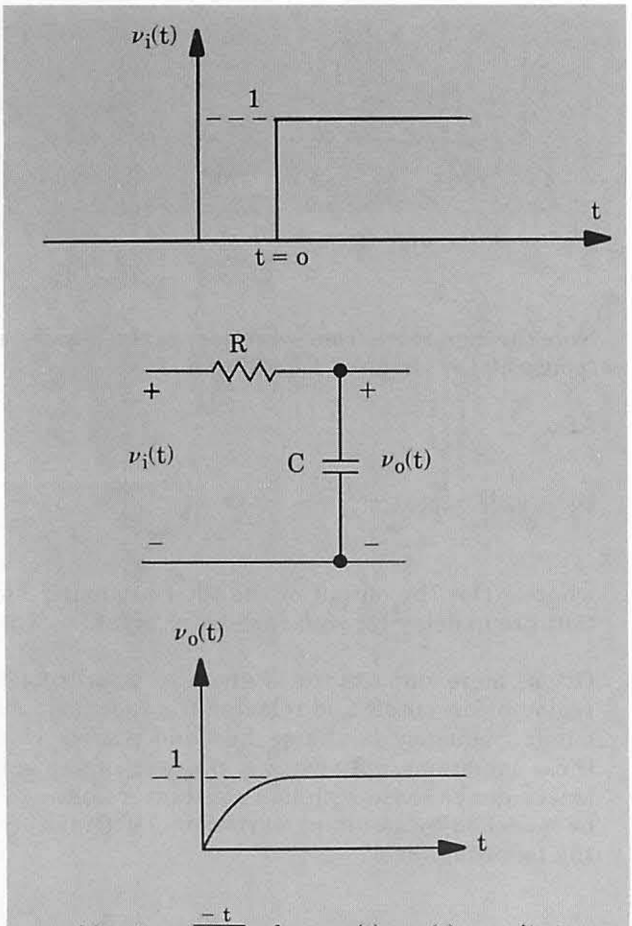
Since phase is generally lagging, group delay is most commonly defined as

$G_d = \dfrac{-d\theta}{d\omega}$ in order to derive a positive number.

III. THE ATTRIBUTES OF PHASE LINEARITY

To demonstrate the beneficial effects of linearizing phase, we present a simple example:

Consider the step response of a simple RC low pass filter as shown in Figure 5.



$\nu_o(t) = 1 - \epsilon^{\frac{-t}{RC}}$ where $\nu_i(t) = \mu(t)$; a unit step

Figure 5. Response of RC Low Pass Filter

The transfer function is given by:

$$H(\omega) = \frac{1}{1 + j\omega RC} = \frac{1}{[1 + (\omega RC)^2]^{1/2}} \epsilon^{-j\tan^{-1}RC}$$

The phase characteristics are given by:

$$\theta(\omega) = -\tan^{-1} RC$$

which is clearly nonlinear as a function of ω as shown in Figure 6.

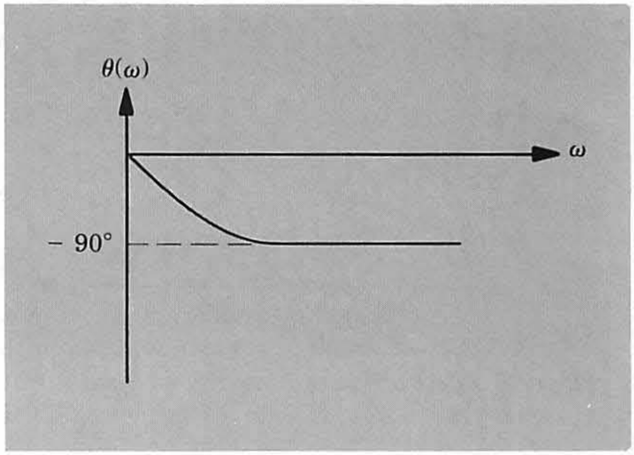


Figure 6. $\theta(\omega) = -\tan^{-1} RC$

Assuming the above amplitude characteristics but with linear phase:

$$H(\omega)_{\text{linear phase}} = \frac{1}{[1 + (\omega RC)^2]^{1/2}} \epsilon^{-j\omega t_o}$$

The response to a unit step $\nu_i(t) = u(t)$ is shown in Figure 7.

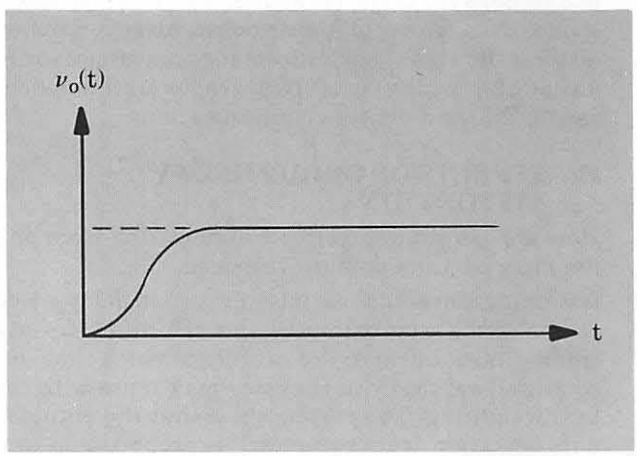


Figure 7. Step Response of Linear Phase RC Low Pass Filter

Note that the phase equalized response is *symmetric* and has approximately 5% faster rise time.

For a square wave input, the resultant outputs are shown in Figure 8.

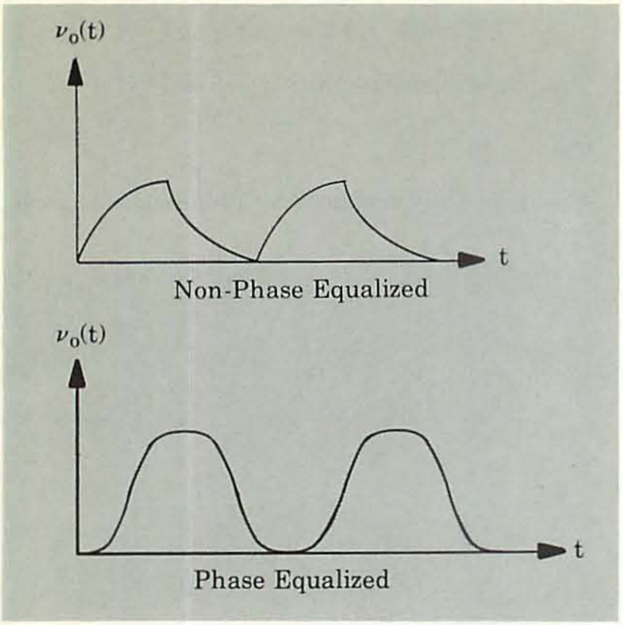


Figure 8. Effect of Phase Linearity on Square Wave

Linear phase is of course, non physical. However, good approximations can be made by phase equalization. While this simple low pass filter is not one we would ever choose for a system, it does demonstrate the importance of linear phase to symmetric, low distortion signal transmission. Certain generalizations can be made based on this example.

In general, amplitude variations or bandwidth-limiting introduces symmetric distortion. Thus, if a pulse is passed through a system with linear phase response, the induced distortion does not interfere with the signal symmetry regardless of the amplitude response. Non-linear phase response, however, not only distorts the signal but distorts it nonsymmetrically—a condition which severely degrades systems such as radar, TV, and digital communications.

IV. EFFECTS OF GROUP DELAY DISTORTION

Here we examine the effects of deviation from phase linearity on time domain response.

Assuming sinusoidal variation in group delay—a condition which approximates the effects of deviation from phase linearity of a filter which has been equalized optimally in the mini-max sense with constant weighting. The results show that the output signals deviation from symmetry is proportional to the amplitude of the variation in group delay.

Let $G_d = \frac{bn\pi}{\omega_c} \cos \frac{n\pi}{\omega_c} \omega + t_o$

where n is an integer and ω_c is some arbitrary cutoff frequency and t_o is some constant delay

integrating:

$$\theta(\omega) = -\omega t - b\sin \frac{n\pi\omega}{\omega_c}$$

and $\Delta\theta(\omega) = -\frac{b}{2}(\epsilon^{jn\pi\omega/\omega_c} - \epsilon^{-jn\pi\omega/\omega_c})$

If $\Delta\theta(\omega)$ is small

then $\epsilon^{-j\Delta\theta(\omega)} \approx 1 - j\Delta\theta(\omega)$

and $H(\omega) = A(\omega)\,\epsilon^{-j\omega t_o}(1 + \frac{b}{2}\epsilon^{jn\pi\omega/\omega_c} - \frac{b}{2}\epsilon^{-jn\pi\omega/\omega_c})$

Let $h_o(t)$ be the impulse response of the filter assuming perfectly linear phase, then

$$h(t) = h_o(t) + \frac{b}{2}h_o(t + \frac{n\pi}{\omega_c}) - \frac{b}{2}h_o(t - \frac{n\pi}{\omega_c})$$

as shown in Figure 9.

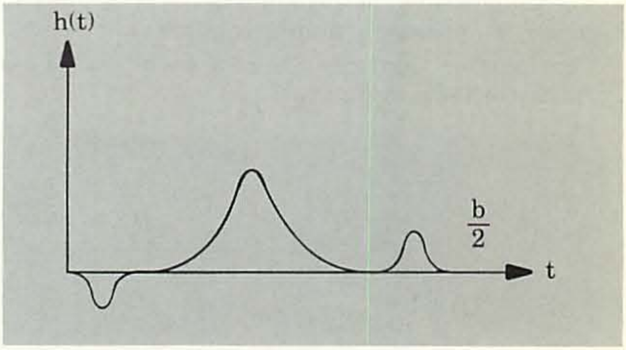


Figure 9. Impulse Response h(t)

Note the departure from symmetry of the impulse response h(t) by virtue of "echos" of $h_o(t)$.

Also,

$$g(t) = g_o(t) - \frac{b}{2}g_o(t + \frac{n\pi}{\omega_c}) + \frac{b}{2}g_o(t - \frac{n\pi}{\omega_c})$$

where g(t) is the output of the filter assuming constant group delay for some arbitrary input.

Often, more importance is given to equalizing the region below cutoff and relaxing the specification as cutoff frequency is approached and passed. Under these conditions, optimization of the equalizer in the least squares sense with non-constant weighting can be modelled by assuming variation of group delay of the following form:

$$-\frac{d\theta}{d\omega} = t_o b\omega \cos \frac{n\pi\omega}{\omega_c}$$

where $\frac{bn\pi}{\omega_c} << 1$ for small distortion;

integrating

$$\theta = -\omega t_o - \frac{b\omega_c}{n\pi}\left[\sin\frac{n\pi\omega}{\omega_c} - \frac{bn\pi\omega}{\omega_c}\cos\frac{n\pi\omega}{\omega_c}\right]$$

and

$$H(\omega) \approx H_o(\omega)\left[1 - \frac{b\omega_c}{2n\pi}\,\epsilon^{\,jn\pi\omega/\omega_c}\,\epsilon^{\,-jn\pi\omega/\omega_c}\right.$$

$$\left. - \frac{j\omega b^2}{2}\left(\epsilon^{\,jn\pi\omega/\omega_c} + \epsilon^{\,-jn\pi\omega/\omega_c}\right)\right]$$

and

$$g(t) = g_o(t) - \frac{b\omega_c}{2n\pi}\left[g_o(t + \frac{n\pi}{\omega_c}) - g_o(t + \frac{n\pi}{\omega_c})\right]$$

$$- \frac{b^2}{2}\left[\frac{dg_o(t - \frac{n\pi}{\omega_c})}{dt} + \frac{dg_o(t + \frac{n\pi}{\omega_c})}{dt}\right]$$

which gives echos of the derivative of g(t) as well as echos of g(t).

V. EQUALIZED FILTERS

There are classes of networks whose group delay variation is extremely small within the network bandwidth. These networks are characterized by rounded amplitude characteristics such as Gaussian. In addition to their rather weak amplitude attributes, it must also be remembered that design tables for these networks are for the *low pass* case and that phase characteristics are not transferrable from the low pass prototype through a band pass transformation except in special cases.

A general method for generating networks with linear phase characteristics are arbitrary amplitude attributes is obviously needed and must utilize non-minimum phase networks.

The most successful method involves the decomposition of network attributes into Amplitude and Variation of Group Delay specifications as shown in Figure 10.

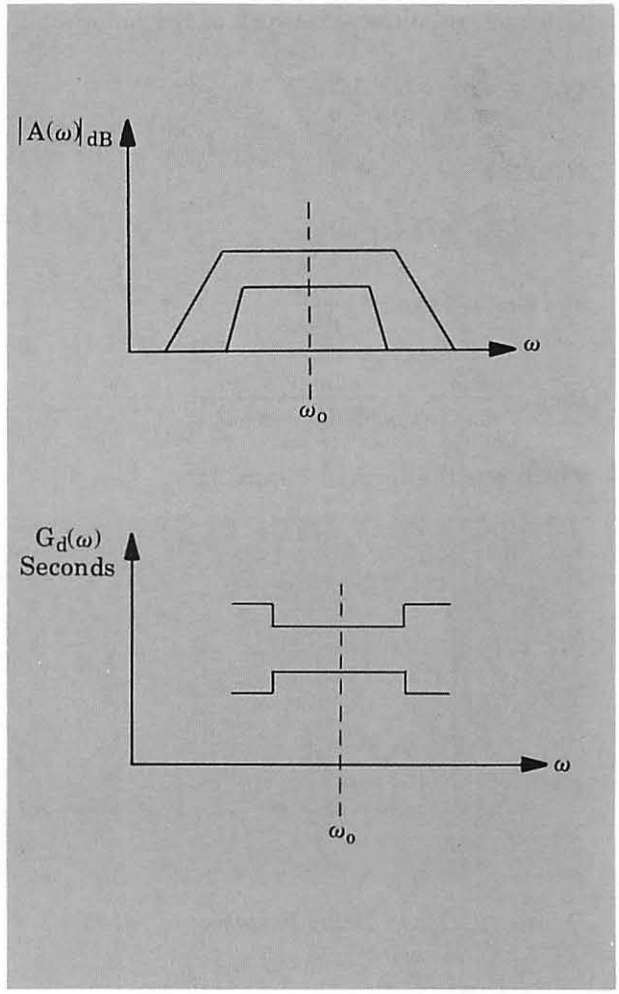


Figure 10. Decomposition of Amplitude and Group Delay Specifications

If we choose a filter which meets the amplitude specification from a design table or indeed synthesize a standard filter such as a low-ripple Tchebbychef, $T(\omega)$, we find that the filter meets the amplitude specification but does not meet the group delay specification as shown in Figure 11.

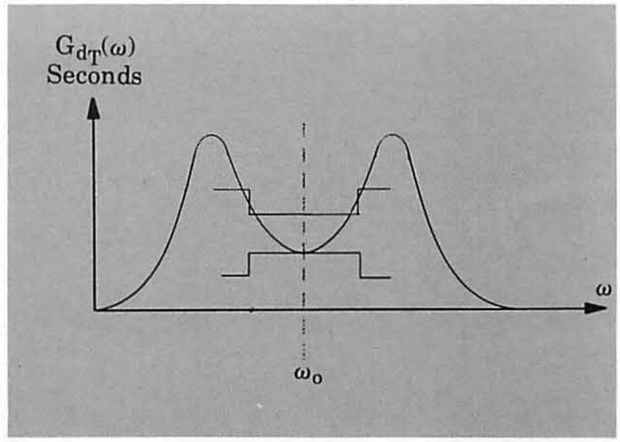


Figure 11. Group Delay Does Not Meet Specification

Consider an all-pass network of the following form:

$$E(\omega) = \frac{(j\omega)^2 - aj\omega + b}{(j\omega)^2 + aj\omega + b}$$

Note that

$$|E(\omega)| = 1 \text{ for all } \omega$$

$$\text{and } \theta(\omega) = 2\tan^{-1}\frac{a\omega}{b-\omega^2}$$

$$\text{then } \frac{-d\theta(\omega)}{d\omega} = \frac{-2a(\omega^2+b)}{(\omega^2-b)^2 + a^2\omega^2}$$

which has the form of Figure 12:

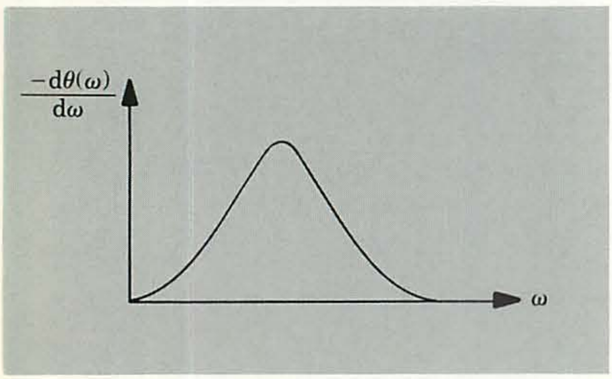


Figure 12. Group Delay Response of All-Pass Network

The total area under the group delay curve for an all-pass network of this form is given by:

$$\int_{-\infty}^{\infty} \frac{-d\theta(\omega)}{d\omega}\, d\omega = 2\pi$$

For concatenated linear networks group delay is additive:

$$A(\omega) = |G(\omega)|\, \epsilon^{j\theta_g(\omega)}\, |H(\omega)|\, \epsilon^{j\theta_h(\omega)}$$

$$|A(\omega)|\, \epsilon^{j\theta_a(\omega)} = |G(\omega)|\,|H(\omega)|\, \epsilon^{j(\theta_g(\omega) + \theta_h(\omega))}$$

$$\text{and } \frac{-d\theta a(\omega)}{d\omega} = \frac{-d}{d\omega}(\theta_g(\omega) + \theta_h(\omega)) = \frac{-d}{d\omega}\theta_g(\omega) + \frac{-d}{d\omega}\theta_h(\omega)$$

$$= G_d \text{ of } G + G_d \text{ of } H$$

By adjusting the parameters a and b and synthesizing $E(\omega)$ as a constant impedance network, $E(\omega)$ can be concatenated with $T(\omega)$ to equalize the variation in group delay *without affecting amplitude response.* Thus Group Delay variation can be equalized as shown in Figure 13, and $E(\omega)$ is called a *Group Delay Equalizer.*

Since all-pass networks can be synthesized as constant impedance networks, additional "sections" can be concatenated as follows:

EQUALIZERS:

$$E(\omega) = \prod_{i=1}^{n} \frac{(j\omega)^2 - a_i(j\omega) + b_i}{(j\omega)^2 + a_i(j\omega) + b_i}$$

$$\text{and } \frac{-d\theta_e(\omega)}{d\omega} = -\sum_{i=1}^{n} \frac{2a(\omega^2 + b_i)}{(\omega^2 - b_i)^2 + a_i^2\omega^2} = G_{de}$$

Note that for n equalization sections there are 2n parameters. Parameter b controls the frequency of the peak while parameter a controls the height of the peak and hence the shape of the curve.

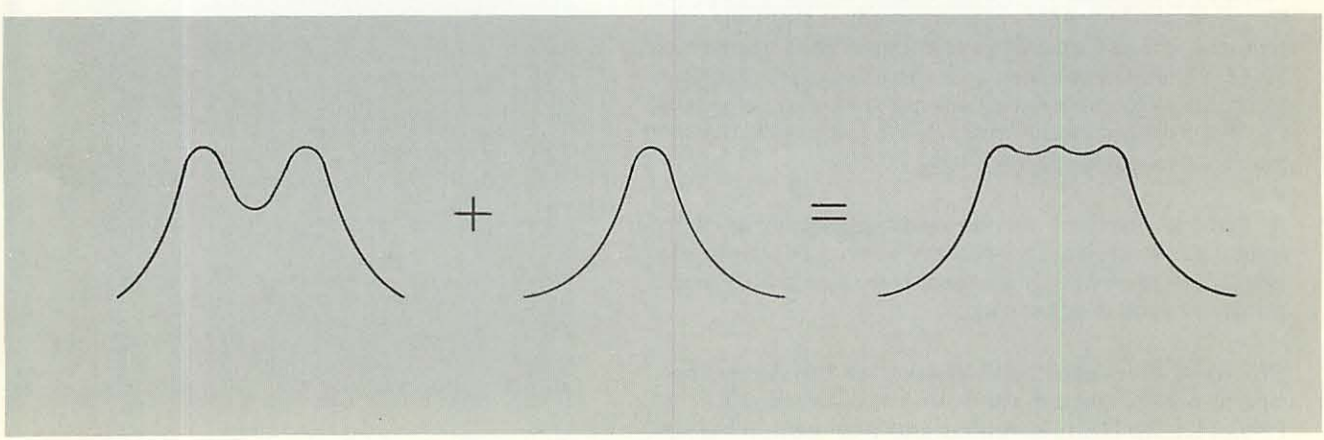


Figure 13. Group Delay Equalization

Calculation of the required parameters is extremely difficult as at present

THERE IS NO ANALYTICAL METHOD FOR DETERMINING THE PARAMETERS
and
GROUP DELAY IS *NOT* SCALEABLE FROM FREQUENCY TO FREQUENCY.

Each filter requirement must, therefore, be custom designed. In addition, each equalizer section affects the group delay curve throughout the frequency range, which is to say, $E(\omega)$ cannot be decomposed for design simplicity—the entire expression must be evaluated for each parameter variation. Needless to say, a great deal of experience is required to effectively design equalizers and equalized filters.

To further complicate matters, true all pass networks cannot be physically synthesized due to parasitic elements and lossy elements. Therefore, variance from the theoretical group delay curve is encountered as well as amplitude variations.

VI. AMPLITUDE RESPONSE OF EQUALIZERS

In Section V, we derived group delay equalizer sections whose amplitude characteristics were all-pass. The physical synthesis of these sections usually takes the form of a non-ladder structure. The singularities are in the form of the quadripole shown in Figure 14.

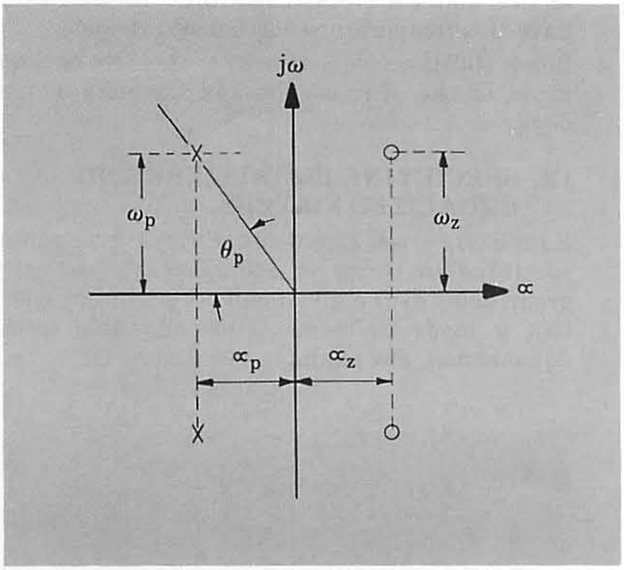


Figure 14. S Plane Representation of All Pass Network

It is apparent that the all-pass characteristics of the quadripole depends upon strict symmetry of the singularities. Physically, the equality of α_p and α_z depends upon the loss characteristics of the components; usually inductors—the higher the Q, the more α_p and α_z approximate equality. In addition, the smaller α_p and α_z are with respect to ω_p and ω_z the higher the Q requirements. These attributes are more easily handled by referring to θ_p.

Define $\xi = \cos \theta_p$

Referring to Figure 15 which shows the amplitude response of an "all-pass" equalizer section

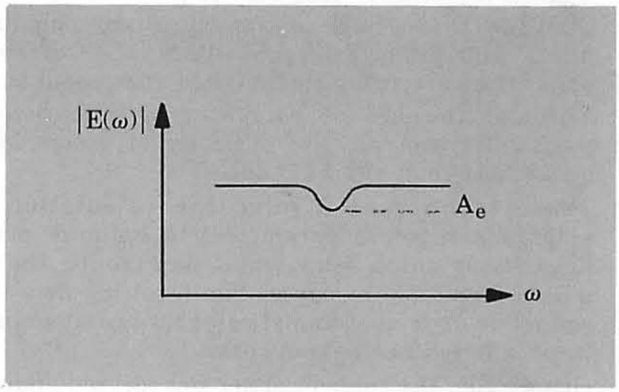


Figure 15. All-Pass Equalizer Section

A_e is approximately given by

$$A_e \approx \frac{2Q\,\xi - 1}{2Q\,\xi + 1} \text{ or in dB}, A_{edB} = 20 \log \frac{2Q\xi - 1}{2Q\xi + 1}$$

where Q refers to the quality factor of the inductors, L_1 and L_2 used in an all pass, constant impedance section of the form of Figure 16.

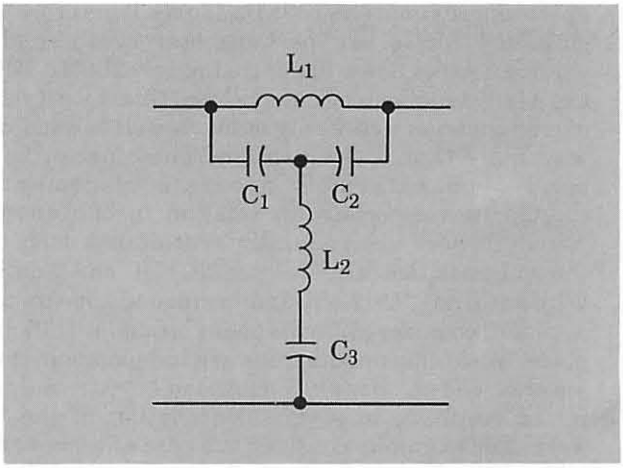


Figure 16. All-Pass Constant Impedance Network

The deviation from true all pass characteristics also changes the form of the group delay curve and must be taken into account when designing group equalizers.

In addition, when utilizing large number of group delay equalizer sections, amplitude equalization may also be necessary as the amplitude bumps of peak deviation A_e add in dB.

VII. COMPUTER OPTIMIZATION

Although group delay equalization can be achieved through hand calculation and trial and error, today's requirements dictate the utilization of computer optimization programs. These programs can achieve least squares and/or least p^{th} approximations to the required group delay specification and can simultaneously optimize amplitude parameters as well as account for parasitic and lossy physical elements. At COMSTRON • SEG, extremely sharp equalized filters with group delay equalizers of 40 or more parameters are regularly designed and produced.

Through the use of computer optimization of amplitude response, the difficulty of group delay equalization can also be reduced.

These techniques involve the calculation of amplitude response parameters to optimize phase linearity or group delay while maintaining the required amplitude response. The resulting networks cannot be arrived at analytically but are superior to classically synthesized networks.

Historically, the filter designer has utilized analytic tools to place singularities judiciously. Most often compromises were made because the lack of computational power and techniques did not permit the designer to efficiently place singularities for optimum results. The requirements of today's communication systems are such that the performance gained through delay equalization and computer optimization is an absolute necessity.

VIII. PASSIVE VS. ACTIVE

Although active networks perform reasonably well up to approximately 100kHz, delay equalizers and equalized filters are perhaps best synthesized as passive devices down to approximately 100Hz. This is especially true for extremely sharp filters with zeroes of transmission in the stop band, as well as equalizers with more than a few sections. These networks depend upon extremely accurate placement of singularities especially in relation to one another. Active devices are generally synthesized such that the singularities are *independent* of one another. While at first glance this independence appears to be a virtue, consider what happens should a shift take place. Since the singularities are independent of one another, the *relative* placement can be extremely distorted resulting in severe degradation of the network. For example, consider the case of an equalizer comprised of many sharp sections as shown in Figure 17.

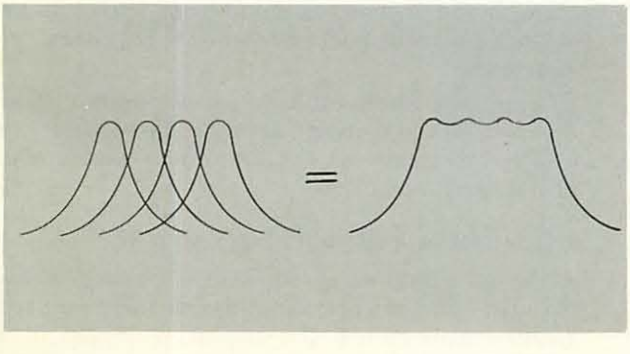


Figure 17. Equalizer

If one element shifts, the result can be as shown in Figure 18.

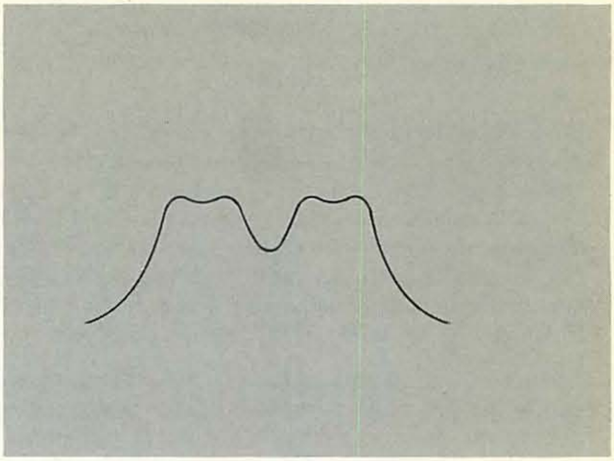


Figure 18. Shift of One Element

Since passive networks are interactive, the *relative* placement of the singularities is reasonably preserved despite a shift of one element. The disastrous effects shown will therefore usually *not* take place with a passive equalizer. In addition, passive networks are more easily temperature compensated because of the cancelling temperature coefficients of ferrite materials and capacitors. Resistors do not have this temperature coefficient attribute.

Below 100Hz active networks are usually dictated because of the size, weight and lossiness of passive devices.

IX. SPECIFYING EQUALIZERS AND EQUALIZED FILTERS

Equalizers and Equalized Filters are generally specificed in terms of the allowable variation in group delay over a given band. Very often, specification is made in terms of an allowable mask for equalization. For example, see Figure 19.

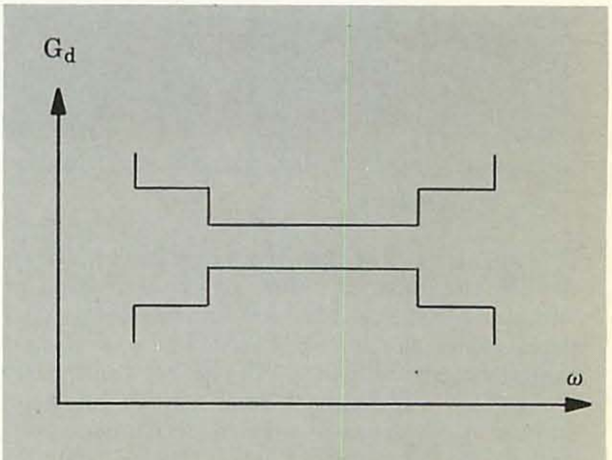


Figure 19. Group Delay Specification Mask

The curve to be equalized can either be specified in terms of a graph or as data points. The *minimum* number of sections required to equalize a curve can be estimated by the area required to fill the curve. For example, referring to Figure 20 the area required to fill the curve from 100Hz is approximately 500μs x 3kHz x 2 π = 2 π (1.5). Since each section can contribute an area of 2 π, a minimum of 1.5 sections is required just to fill up the required area. In order to match the curve, additional sections are required so an estimate of three or four sections is in order assuming the tolerance required is not stringent.

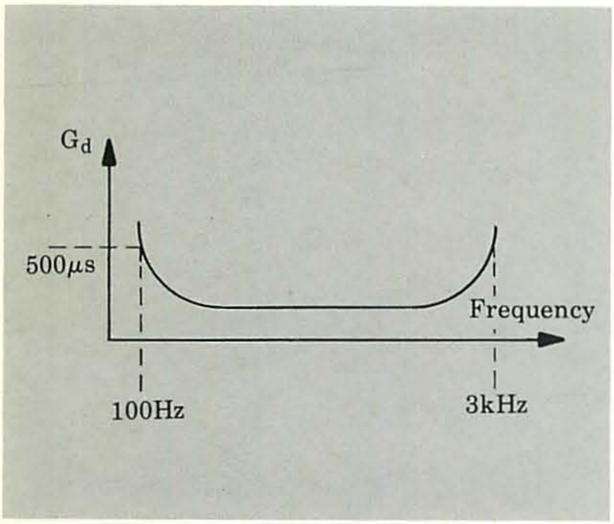


Figure 20. Area to Fill Group Delay Curve

It is important to understand that the absolute delay within a circuit cannot be reduced. Equalizers correct, at a specific point in the circuit, for relative values of delay distortion across the passband. Fundamentally, relative delay distortion is corrected by adding extra delay to those areas in frequency where the delay value is low. This effectively flattens the passband by setting the overall distortion levels across the band, at the value of the highest delay. Even though the absolute (total) delay has been increased, distortion due to variation in group delay is substantially reduced.

The best *tolerance* to which equalization can generally be achieved is approximately 1% if the *total delay of the equalizer.* This tolerance generally takes the form of ripples superimposed on constant group delay. In the above case, the total delay of the equalizer would be approximately 1000μs and the tolerance would be 10μs. In order to achieve this tolerance, however, many more sections would be required. The maximum number of sections that generally can be realistically used is thirty.

It should be remembered that smooth shallow curves are easier to equalize than bumpy or steep curves. Wideband equalizers sometimes present added difficulty because of the large amount of area required to be filled and the potentially wide range of component values needed to maintain constant impedance. In addition, good equalization is generally more important at midband or midchannel than at band edges. Therefore, equalizer specifications are usually drawn to be most stringent at mid-band and relaxed at band edges.

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2. Communication Systems and Techniques. Authors; Schwartz, Bennet, and Stein, McGraw-Hill, 1966.

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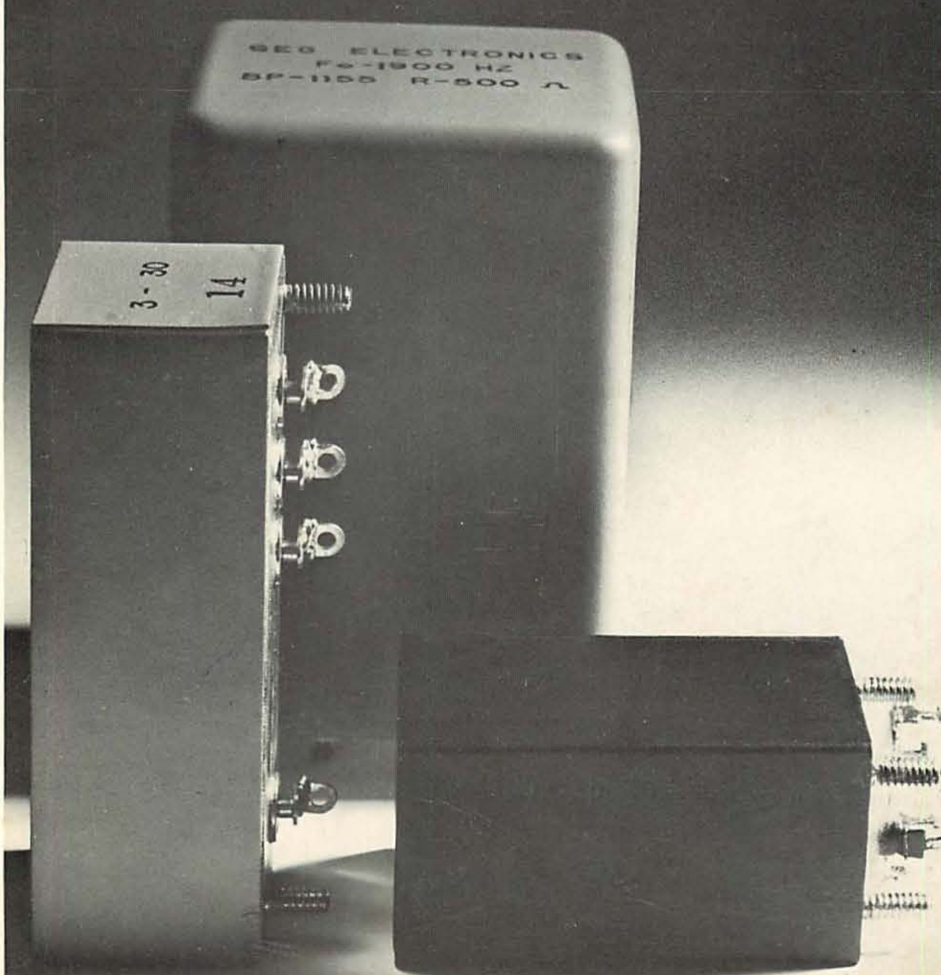
- Low Pass
- Band Pass
- Band Reject
- Matched
- Delay Equalized
- Amplitude Equalized
- Time Domain
- FDM
- Optimum
- Thompson
- Butterworth
- Chebyschev
- Cauer
- Bessel
- Elliptic
- Legendre
- Channel Simulation



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